Asynchronous Models For Consensus

Lecture 5

Further reading:

Distributed Algorithms
Nancy Lynch,

Distributed Consensus

**Problem 1** - Consensus, synchronous settings, unreliable communication: impossible.

**Problem 2** - Consensus, asynchronous settings, unreliable communication: impossible

(Problem 1 is a special case of Problem 2).
The Asynchronous Model

- Asynchronous setting
- Complete network graph
- **Reliable FIFO unicast** communication
- Deterministic processes, \{0,1\} initial values
- **Fail-stop failures of processes are possible**
  (remember that this is solvable in a synchronous setting)

Solution Requirements for Consensus

- **Agreement:** All correct processes decide on the same value
- **Validity:** If a correct process decides on a value, then there was a process that started with that value
- **Termination:** All processes that do not fail eventually decide
Impossibility Result (FLP[85])

Definitions:
- **x-fair** execution: executions in which all channels execute fairly, and all processes but at-most x execute fairly
- **0-RCP** : (0-resilient consensus protocol) - a protocol that solves consensus in all 0-fair executions
- **1-RCP**: a protocol that solves consensus in all 0-fair and 1-fair executions

FLP[85] (Cont.)

**FLP**: There is no 1-Resilient Consensus Protocol!

Question 1: Can you think of a 0-Resilient Consensus Protocol?

Question 2: what can be problematic if one of the processes may crash?
More Definitions...

- A finite execution $\alpha$ is **0-valent** if 0 is the only decision value in all extensions of $\alpha$.
- A finite execution $\alpha$ is **1-valent** if 1 is the only decision value in all extensions of $\alpha$.
- $\alpha$ is **bivalent** if it is neither 0-valent nor 1-valent.

Lemma 1:

**In any 1-Resilient Consensus Protocol there is a bivalent initial execution**

Proof of Lemma 1

- If $(i_1, i_2, \ldots, i_n) = (0, 0, \ldots, 0)$ => decision is 0
- If $(i_1, i_2, \ldots, i_n) = (1, 1, \ldots, 1)$ => decision is 1
- Assume that each vector $(i_1, i_2, \ldots, i_n)$ is univalent
- Look at: $(0, 0, \ldots, 0, 0), (1, 0, \ldots, 0, 0), (1, 1, \ldots, 0, 0), \ldots, (1, 1, \ldots, 1, 0), (1, 1, \ldots, 1, 1)$
- from all the above, there exists two starting vectors that are identical except for one entry of some processor $p$, where $v_0$ is 0-valent and $v_1$ is 1-valent
- Kill $p$ at the beginning to reach a contradiction
A Decider

A Decider for algorithm A consists of execution $\alpha$ of algorithm A and a process $p$ such that:

- Execution $\alpha$ is bivalent.
- There exists 0-valent extension $\alpha 0$ of $\alpha$ such that the suffix after $\alpha$ consists of steps of $p$ only.
- There exists 1-valent extension $\alpha 1$ of $\alpha$ such that the suffix after $\alpha$ consists of steps of $p$ only.

Illustration of a decider

- $p$ may receive a message and then send a message or send a message and then receive a message
- Alternatively $p$ may receive 2 messages at different orders
Correctness of FLP

Lemma 2:
Let A be a 1-RCP with a bivalent initial execution. There exists a decider for A.

-- FLP is correct if Lemmas 1 and 2 are correct:
Why?

Lemma 1: In any 1-RCP there is a bivalent initial execution

Together they mean that:
in any 1-RCP there exists a decider.

Correctness of FLP (Cont.)

-- FLP is correct if Lemmas 1 and 2 are correct:

Note: only p moves in α₀, α₁
Proof of Lemma 2

For 1-RCP, we can delay messages from one process and still expect termination (!)

Suppose that after $\alpha$, a bivalent execution, the delivery of $m$ to $p$ yields a univalent execution. WLG assume it yields 0-valent.

Proof of Lemma 2 (cont.)

To reach a 1-valent extension of $\alpha$ there are two possibilities:

1. $m$ is not delivered before decision is reached
2. $m$ is delivered somewhere before decision is reached

In the first case, we deliver $m$ after the decision is reached (i.e. after reaching a 1-valent execution.)
Proof of Lemma 2 (cont.)

Case 1

\[ \alpha \]

m delivered (to p)

0-valent

1-valent

m delivered

Case 2

\[ \alpha \]

m delivered

0-valent

bivalent

m delivered

1/0-valent

0/1-valent

In case 2, pick another message \( m' \) further down. This going down process has to be finite because of termination.

Proof of Lemma 2 (end)

We stick the delivery of \( m \) after each step (look at Case 1)

\[ \alpha \]

m delivered (to p)

0-valent

0-valent

0-valent

1-valent

1-valent

1-valent

This is a decider!

There has to be a step which before it we have 0-valent and after 1-valent.

This step has to be made by \( p \).
So, What can be done???

We need to pay something in order to gain something else.

What can we pay? (what can we gain?)

A Randomize Protocol for Consensus

A complete network graph (clique)
n - total number of processes.
f - total number of faulty processes.
Assumption: \( n > 5f \).

This algorithm solves a more complex problem where the failure model is Byzantine, i.e. the failed processes can send arbitrary messages to arbitrary processes (may lie), or may fail.
The protocol (Ben-Or variation)

Round=0; x = initial value
Do Forever:
    Round = Round + 1
    Step 1
    Step 2

Step 1:
    Send Proposal(Round,x) to all processes
    wait for n-f messages of type Proposal(Round,*)
    if at least n-2f messages have the same value v
        then x = v (that value)
        else x = undefined

Step 2:
    Send Bid(Round,x) to all processes
    wait for n-f messages of type Bid(Round,*)
    v is the real value (0/1) occurring most often
    and m is the number of occurrences of v
    if m >= 3f+1
        then Decide (x=v)
    else if m >= f+1
        then x = v
    else x = random (0 or 1)
Other Ways to Bypass The Impossibility Result

• To allow the protocol not to guarantee agreement.

• To allow the protocol not to always terminate at all correct members:
  – The Transis membership can exclude live but “slow” processes from the membership, and will reach “agreement” among the connected members.