ABSTRACT

The detection of hazardous radiation sources is difficult for several reasons. Sensors detect when photons, generated randomly by sources, strike crystals and so sensors don’t get continuous information streams. Radiation emanates from threats but also from walls, the ground and medical patients undergoing radiation treatment. There may be multiple, mobile, evasive sources. The number of parameters is so large that fundamental tradeoffs in designing sensor networks for the problem are unclear. This paper reports on analyses of basic tradeoffs such as mobile versus static sensors; few large sensors versus several small ones; unlimited communication range and bandwidth versus limited range and bandwidth between sensors. The paper restricts attention to detecting sources in a two-dimensional area. Understanding basic tradeoffs - such as the reduction in detection time that accrues from increased speed of mobile sensors within a 2D field - helps in designing systems for more complex settings such as urban areas with buildings and roads.

General Terms

Algorithms, Simulations

Keywords

Radiation detection, wireless sensor network, mobile sensors, distributed data processing, detection and tracking

1. INTRODUCTION

Intelligent Personal Radiation Locators (IPRLs) have been developed recently to detect hazardous radiation sources. These devices have small form factors and are carried by security personnel or on mobile robots. IPRLs communicate with each other and with information fusion stations through wireless or, in the case of tethered devices, through wired connections. Intelligent radiation sensor systems (IRSS) have been proposed to integrate information from multiple detectors in real time so as to improve the probability of detecting hazards, reduce rates of false positives, localize hazardous sources more quickly, and differentiate threats from naturally occurring radioactive material (NORM). This paper studies basic tradeoffs in IRSS. Next, some of the fundamental questions that arise in IRSS designs are discussed briefly; details are provided later.

The core of an IPRL is a plate of material that generates current when struck by a photon. Larger plates are more sensitive because they are more likely to be hit by photons, but they are also more expensive. What are tradeoffs between fewer, more expensive, more sensitive sensors and greater numbers of less expensive, less sensitive sensors? How do these tradeoffs change when sensors can or cannot communicate with each other? What are good designs for using combinations of stationary and mobile sensors? Should mobile sensors search spaces independently and meet periodically to exchange and fuse data, or should they attempt to stay within communication range of each other at all times? This paper studies such questions using a mathematical model based on Bayesian statistics. Parameters of the study are based on existing and future IPRLs.

Prevention of illicit trafficking of fissionable nuclear devices is a major concern for humanity in general and the Department of Homeland Security in particular [10]. The current solution to the detection of these devices involve the use of individual, large, portal-monitor-style detectors positioned at gate points [11]. Research suggests that IRSS are cheaper, more power efficient and enable more rapid deployment than portal monitors [12]. Some earlier work suggests diminishing marginal returns from increasing the size of detectors because the signal-to-noise ratio (SNR) does not improve proportionately...
to surface area \[^{14}\]. In this paper we study basic problems, using simple models and easily reproducible experiments to gain insight into design tradeoffs.

The major source of error in radiation detection is background radiation \[^{5\ 14\ 11\ 6}\]. Background radiation is ionizing radiation emitted from a variety of natural and artificial sources. Primary contributions come from the earth’s surface, sky, buildings, and people. The level of background radiation varies with environmental conditions such as rain and snow. Patients who have received radiological treatment such as Technetium-99 or Iodine-131 also emanate photons \[^{12}\]. More expensive sensors can distinguish harmful radiation sources from benign sources by their energy spectrum. One of the many tradeoffs to be considered are the relative number of expensive sensors that identify source signatures accurately and the number of less expensive sensors that do not.

IRSS are useful in a variety of settings including the detection of dirty bombers infiltrating political rallies while carrying radiation material in backpacks \[^{1}\]; detection of nuclear material on ships by boarding parties \[^{2}\]; and detection of radioactive contraband at airports \[^{3}\]. Models of realistic scenarios, such as detection by maritime boarding parties carrying sensors, are complex; they have to deal with shielding by metal shields such as ship hulls and evasive action likely to be taken by smugglers. We study the simpler problem of detecting static sources in a 2-dimensional region so as to gain fundamental insights into tradeoffs.

We consider the scenario in which radioactive materials, such as Cesium-137 or Cobalt-60 are carried into a 2-D field and then left in place or are moved slowly by pedestrian terrorists. Sensors have wireless communication capability, as well as GPS or some other location determination mechanism. The system architectures we consider include combinations of static and handheld detectors. We study tradeoffs between wireless sensors with long communication ranges that allow mobile sensors to remain in communication contact much of the time and sensors with short ranges which reduce opportunities for information aggregation.

2. DETECTOR MODEL

The traditional Geiger counter is limited by its long dead time and large size \[^{10}\]. A collaborative effort between Caltech, Smith Technology, and Motorola is building an IPRL based on a Cadmium Zinc Telluride (CdZnTe) detector coupled with GPS, gyroscope, and wireless capacity.

The CdZnTe detector was introduced only in the last decade and has shown great improvement in both size and energy resolution over other room temperature operated gamma-ray spectrometers, such as NaI scintillator. The technology with CdZnTe has also made compact, light-weight detector systems possible \[^{7\ 5}\]. The prototype of the IPRL detector consists of a 20mm × 20mm × 5mm slab of CdZnTe crystal which is pixelated into an array of 32 × 32 smaller detectors, each with smaller SNR than a single readout of the full detector. The detector and the platform it is mounted on is lightweight, compact, inexpensive, and potentially suitable for carrying onto small ground robots and UAVs. Figure 1 shows a model of the IPRL detector.

![Figure 1: IPRL handheld detector](image)

The current specification of the wireless/GPS module is listed below.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Requirement</th>
<th>Goal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location (GPS)</td>
<td>&lt;5m, latitude, longitude &lt;5m</td>
<td></td>
</tr>
<tr>
<td>Wireless Range (peer-peer)</td>
<td>&gt;20m</td>
<td>&gt;200m</td>
</tr>
<tr>
<td>Wireless Range (peer-master)</td>
<td>&gt;300m</td>
<td>&gt;5km</td>
</tr>
</tbody>
</table>

3. RADIATION MODEL

3.1 Photon Emission from Radiation Source

Photons are generated by a radiation source in a Poisson manner. Consider a fixed radiation source and a fixed detector where the rate at which the detector records photons from the source is \(\lambda\). Then the probability of the detector recording \(n\) hits in time interval \(\Delta t\) given intensity \(\lambda\) is

\[
P(n \mid \lambda, \Delta t) = \frac{(\lambda \Delta t)^n \cdot e^{-\lambda \Delta t}}{n!}
\]

We approximate a radiation source as a point source. This approximation is reasonable when detectors are far from the source. We also assume that photons are sent
with equal intensity in all directions, that is, the source is shielded evenly. The detector crystal is approximated to be a flat plate with surface area \( A \) and arbitrarily small thickness. These approximations help us gain insight into basic tradeoffs such as those between many large sensors and few small ones.

Let \( \mu \) be the rate at which photons are generated by the source. Let \( d \) be the distance between the source and the detector. The rate at which photons pass through a surface area \( A \) on the surface of a sphere of radius \( d \) is proportional to \( \frac{1}{d^2} \). The rate \( \lambda \) at which photons are detected by the sensor is given by:

\[
\lambda = \frac{\mu \cdot A \cdot \cos \theta}{d^2}
\]  

(2)

Where \( \theta \) is the angle between the incoming photon direction and the normal to the surface it hits. This equation is only valid when the distance \( d \) from the source is large. For the parameters of interest, this equation is a satisfactory approximation when \( d \) is of the order of a few meters. In the limiting case, as \( d \) approaches zero, \( \lambda \) approaches \( \mu \); and we use this correction factor to model the small \( d \) case.

Photon Absorption.

Equation (2) does not consider absorption of photons by material on the path between the source and detector. Absorption is calculated as follows. We consider the ray from the source to a point on the detector surface and determine the material and its density along that ray. We then calculate the probability of absorption of photons along that ray, and adjust \( \lambda \) accordingly.

Simulating Photon Stream.

When the sensor and the source move, the values of \( d \) and \( \theta \) change with time. In this case \( \lambda \) is time varying and the process by which the sensor detects photons is an inhomogeneous Poisson process. We handle the time-varying nature of the process by carrying out calculations in time increments and assuming that the Poisson intensity does not vary within each increment. Using Equation (1) the probability of a detector with intensity \( \lambda \) receiving at least one photon hit in time \( \Delta t \) is:

\[
P(n > 0 \mid \lambda, \Delta t) = 1 - e^{-\lambda \Delta t}
\]

(3)

This equation is accurate provided the time increment \( \Delta t \) is small relative to the linear velocities of the source and sensor and the angular velocity of the sensor. In scenarios such as maritime boarding or patrolling political rallies, IPRLs are carried by security personnel. Changes in orientation when a person turns or looks down changes \( \theta \); We approximate that \( \lambda \) does not change during the interval when \( \Delta t \) is small enough. We cannot make \( \Delta t \) arbitrarily small, however, because of the **dead time** of the sensor. The **dead time** is the minimum time that must elapse between consecutive hits in order for them to register as unique events [5]. The IPRL has a dead time of about 2 milliseconds, which is \( \frac{1}{10} \) of the \( \Delta t \) used in our simulation. We chose this \( \Delta t \) to conserve the processing power. This approximation is reasonable because in real scenarios, we are likely to be far away from the source and frequent updates are unnecessary.

3.2 Background Radiation

Contributions from cosmic and atmospheric radiations do not vary much in a small area and are modeled with a fixed probability of photon hits at the detector. Contributions of photons from material such as rocks and bricks do vary from detector location to location. Though our mathematical model accounts for background radiation from material, we do not consider them in the experiments reported in this paper except the very last one because fundamental insights into basic tradeoffs are clouded by consideration of too many parameters such as location-dependent background radiation. Therefore, background radiation from material and cosmic rays is assumed to exist but to be uniform at all points in the area.

4. LOCATION ESTIMATION

We describe problems in order of increasing realism and complexity. We begin by considering the problem of detecting a single source and later consider multiple sources. We first consider static sources and later discuss moving sources. We begin by assuming that the intensity of the source is known and then consider the more realistic case where source intensity is unknown. In practice, the intensity of a hazardous source is unknown because the source may be shielded or the amount of hazardous material is not known. Nevertheless, simplifying assumptions help us gain insight into basic tradeoffs.

We use Bayesian methods to calculate the probability that the source is at each point in the region. A challenge is to calculate these probabilities quickly. We assume that each IPRL has computational capability or is connected to a handheld computer. The granularity of calculations can be adjusted to suit available computational power.

The region to be searched is partitioned into incremental areas. A simple partition is an equally-spaced rectangular grid. This approach has the problem of either too many grid points or grid sizes that are too large. Later we consider dynamic gridding where the sizes of grid elements are modified over time so that the total number of grid elements is modest and where ar-
eas with parameters changing rapidly over space have smaller grid elements.

4.1 Single Static Source with Known Intensity

In this scenario we consider the case where a single source, with known intensity $\mu$, is known to exist somewhere in the region. The problem is only to detect where the source is. This contrived scenario is useful for analyzing realistic situations.

Consider a uniform grid over the search space. Let the grid elements be indexed $k$ for $k = 1, \ldots, K$. Let the probability that the source is present in a grid element $k$ at time $t$ be $p^0_k$ for $1 \leq k \leq K$, and $t = 0, 1, \ldots$. Since we are certain that there is exactly one source in the region, the sum of probabilities for the region is unity.

$$\forall t : \sum_{k=1}^{K} p^0_k = 1 \quad (4)$$

We begin with an a priori distribution of values $p^0_k$. For our experiments we assume a uniform prior distribution over grid elements. For each time step $t = 1, 2, \ldots$, we compute the a posteriori distribution $p^t_k$ for all $k$, given the sensor readings between times $t-1$ and $t$, and the prior distribution $p^{t-1}_k$ for all $k$. The duration of each time step is a constant $\Delta t$.

Consider a scenario with $J$ sensors indexed $j = 1, \ldots, J$, where $J > 0$. Let $\lambda_{j,k}^t$ be the rate at which photons from a source at grid point $k$ strike sensor $j$. This rate depends on the location and orientation of sensor $j$ as specified in Equation 3. We compute the probability that the source is located at each grid point given the trajectories of all $J$ sensors. This calculation assumes that all sensors can communicate with each other or to a base station. Later, we compare these results with the case where sensor communication bandwidth and range are limited.

Let $f(n^t_j|j,k, \Delta t)$ be the conditional probability of $n^t_j$ photons striking sensor $j$ during the $t$-th time step given that the source is at grid element $k$. Then, from Bayes Law,

$$\frac{p^t_k}{p^0_k} = C \prod_{j=1}^{J} f(n^t_j|j,k, \Delta t) \quad (5)$$

where $C$ is a constant of proportionality computed from Equation 4.

We display $p^t_k$ as gradient of colors on a map, also known as heat map (Figure 2).

We assume that the duration of each time step is small enough that the probability of two photons striking a sensor in a single tick is very small and can be ignored. Thus, during each time step, a sensor detects either no photons or exactly one photon. Then,

$$f(n|j,k, \Delta t) = \begin{cases} e^{-\lambda_{j,k}^t \Delta t} & \text{if } n = 0 \\ 1 - e^{-\lambda_{j,k}^t \Delta t} & \text{if } n > 0 \end{cases} \quad (6)$$

Communication Bandwidth.

Consider the bandwidth required for all $J$ sensors to exchange information. During each update a sensor has to convey its position ($x$ and $y$ coordinates), orientation ($x$, $y$, and $z$ coordinates) and a single bit indicating whether it received a photon or not. The five real-number coordinate values require much more bandwidth than the single bit that indicates the presence or absence of a photon. If, however, the sensors are static or if they are moving along pre-arranged trajectories then coordinate values of all sensors are known beforehand.

4.2 Unknown Intensity

The Bayesian update method for location estimation is limited in that it requires knowing all the parameters in Equation 3. In Equation 5, $\lambda_j^t$ is measured and $\Delta t$ is approximated. However, $\lambda_{j,k}^t$ is related to the intensity of the source $\mu$, which is often unknown in a real world situation (e.g. a dirty bomb threat). A close estimate of source intensity is crucial to estimate the source location. In this section, we present three methods for acquiring an estimate of source location without knowing the source intensity.

4.2.1 Parametric Update Using A Priori Distribution of Intensity

We introduce another dimension into the original space of uncertainty. The result is a three-dimensional estimation problem with the dimensions of consideration being $x$, $y$, $\mu$. We then apply the Bayesian method over this three dimensional space given an a priori distribution of $\mu$ till we reach convergence in both location and intensity. The detailed steps are listed in Appendix A.

4.2.2 Non-Parametric Update in A Sensor Grid

The parametric update method is powerful but computationally intensive. Experimental results have also shown that the choice of the a priori distribution of source intensity is critical to the quality of the predictions. In fact, an ill-posed a priori distribution is worse than no distribution at all. Therefore, the second method for intensity estimation is to neglect $\mu$ completely in Equation 5 and update the probability based only on the relative distance to the source. More information is included in Appendix B. Though simple to apply, this method does not have the same accuracy as the other two methods.

4.2.3 Maximum Likelihood Estimator Method
The source intensity and location can be estimated at the same time using the Maximum Likelihood Estimator (MLE) given that we can log the readings from the sensor, GPS, and gyroscope for each detector at each update. The detailed method and derivation are given in Appendix C.

4.3 Mobile Source

The position of a source moving along a straight line can be easily estimated using the Bayesian method with an array of detectors on the side [11]. However, this method fails when source movement is random [13]. We track a slow moving source by applying a smoothing box filter at the "hot spot" by spreading the probability of the most probable location to its neighbors. Our result shows that this simple method is robust for speeds up to human running pace. More elaborate method such as Kalman Filter or Particle Filter can be applied. This scenario can be simplified given information of the environment that can help predict the movement of the source [9].

4.4 Multiple Sources

The conditional probability calculated from Equation 5 assumes the presence of exactly one source in the area of interest. Multiple sources can thus be identified in a one-at-a-time manner. Our experiments show that sensors locate sources near them without too much interference from distant sensors. After a source is identified, (i.e. when a certain percentage of probability is concentrated in a small area), it is marked as found at the position and the conditional probability is recalculated given this information.

4.5 Effect of Noise

The noise in sensor readings comes mainly from background radiation. The amount of background radiation can be coarsely modeled as a uniform increase in detector intensity. This information can be collected beforehand by monitoring the environment with detectors over a long period of time. If this cannot be done, we can still estimate the background intensity based on the types of objects, such as brick, that are present. The influence from background radiation is considered when calculating the detector intensity $A$ using Equation 2. The background noise is generally much weaker than the radiation source and does not affect the collaborative result of multiple detectors.

Other significant sources are patients who recently received radiological treatment. In this case, the trace radiation is strong enough to affect the outcome of the detection (i.e. a false positive). This problem is unavoidable unless we can examine the spectrum of the radiation. The current solution is to treat it as another radiation source and rule out its influence after it is identified using the algorithm described in Section 4.4.

5. DETECTION ALGORITHMS

The Bayesian update method described in Section 4 allows us to partition the field of interest into the more probable area and the less probable area. This section illustrates three algorithms to efficiently explore these areas based on the constraints given. We first consider the simplified case where there is no restriction on wireless capacity. This discussion serves as a stepping stone when we discuss the realistic case where detectors have limited range and bandwidth.

5.1 Unlimited Range and Bandwidth

At each update ($\Delta t = 0.02$), all detectors broadcast messages of new data obtained since the last update. The message consists of the current position of the detector, the direction it is facing, and whether it receives a hit or not in the last $\Delta t$. The packet takes the following format.

<table>
<thead>
<tr>
<th>x_pos</th>
<th>y_pos</th>
<th>x_fac</th>
<th>y_fac</th>
<th>z_fac</th>
<th>0/1</th>
</tr>
</thead>
</table>

The detectors update their local a posteriori distribution based on all the information received. Since all detectors can communicate with each other, their local maps should be the synchronous view of the global map.

Algorithm 1.

Each detector then moves towards the hottest spot (area with the highest probability) $x_{hot}$ on its map independently. The trajectory is not a straight line from its current position to $x_{hot}$ but a spiral that maximizes the sum of probabilities on its way to $x_{hot}$. It is sufficient that the detector stays at a distance of 4 meters away from $x_{hot}$ because little is gained by moving closer. If the source is real, the detector marks the spot as a found source and notifies all other sensors in the network.

5.2 Limited Range and Bandwidth

Based on the specification in Table 2, we assume the wireless communication is limited by 100 bytes/sec and 100 meters for peer-to-peer message exchange. To meet these constraints, we first reduce the update rate from every 0.02 seconds to every 0.3 seconds. We can compensate for the loss of information between updates by changing the packet format to include photon hits since the last updates. Other sensor can use this information to extrapolate the trajectory of a certain detector since the last update. The new packet becomes:
### Table 3: Packet Format (21 bytes + 15 bit)

<table>
<thead>
<tr>
<th>1 byte</th>
<th>8 bytes</th>
<th>12 bytes</th>
<th>15 bit</th>
</tr>
</thead>
<tbody>
<tr>
<td>ID ((x_{pos}, y_{pos}))</td>
<td>((x_{fac}, y_{fac}, z_{fac}))</td>
<td>((0/1)^{15})</td>
<td></td>
</tr>
</tbody>
</table>

The extrapolation works well when the detectors are held by walking security personnel. When the detectors are moving at a high speed, such as on a UAV, we risk introducing incorrect information. In the scenario we consider, the detectors always move at a slow speed.

A detector \(D_i\) still broadcasts messages in every update period. But now under the 100 meter range constraint, only a subgroup of detectors near \(D_i\) can receive these packets and use them to update their maps. Clearly, each detector now only has part of the global view. From now on, two algorithms are used for detector motion.

**Algorithm 2.**

The detector still moves independently towards the hottest spot in its local map using the same strategy as in Algorithm 1. This algorithm requires almost no change to Algorithm 1, except a check on the range.

**Algorithm 3.**

Detectors within range of each other first reach consensus on the hot spot before moving towards it. This consensus is achieved through extra message exchanges of each’s hot spot and the probability associated with it. The consensus is the hot spot with the highest probability in the group. This algorithm causes detectors to stay together once they enter the range of others.

### 6. EXPERIMENTS

#### 6.1 Simulation Platforms

We model the radiation detection process on the following two platforms:

1. **Java and Matlab** [4]: The physics of radiation emission and the Bayesian update algorithm are built in Java. The flat file output is post-processed in Matlab for graphical representation (Figure 2(a)).

2. **Half-Life 2 game engine**: We extend the Half-Life 2 game engine by incorporating the physics of radiation. The extended game engine allows us to model the more complex properties of radiation, including photon absorption through obstacles and background radiation from environmental objects [8]. We are also able to build and test different detection algorithms in very realistic environments (Figure 2(b)).

#### 6.2 Parameters

The following parameters are used in the experiments. We assume known source intensity in all experiments. Background radiation is not considered except in the last experiment, where we use the value below in our calculation.

- Detector surface area \((A) = 25\text{cm}^2\)
- Detector model = spherical
- Radiation intensity \((\mu) = 10^7\text{photons/second}\)
- Background radiation = 1 photon every 10 seconds

We model the detector as spherical in our simulation (i.e. it receives photons uniformly from all directions). This model can be physically built using a combination of multiple IPRL detectors. This abstract model allows us to focus on the detection algorithm.

#### 6.3 Goals and Metrics

The major goal of these experiments is to compare tradeoffs of various design choices when building a wireless sensor network for radiation detection. We use two metrics for evaluation.

- **Time to convergence**: This metric gives us the measure of how efficient the system operate under certain configuration. This is given by the amount of time it takes before it reaches the stopping criteria as listed below.

  - **Robustness**: This metric characterizes how well the system responds to small variations such as changes in source location.

The source is considered found when the specified stopping criteria is satisfied.
• **Criteria 1**: 50% of the probability is concentrated in 1% of the area of the field.

• **Criteria 2**: 95% of the probability is concentrated in 1% of the area of the field.

### 6.4 Formations of Static Detectors

We start by asking the question, "How should we deploy a collection of static detectors?". In this section, we present two types of formations — **Line**, and **Square** — and compare the result based on the assumptions that the detector can communicate with unlimited range and bandwidth. These experiments provide insight into situations where sensors are arranged randomly with the straight line and square being extreme cases because the square formation is desirable and the straight line is poor. The results are evaluated using Criteria 1. Each data point is the average of 50 independent trials.

#### 6.4.1 Line

We consider a line of nine spherical detectors, each 30 meters apart, on one side of a 300 $\times$ 300 meters field, 30 meters from each edge. A radiation source is placed at a distance $L$ perpendicular from the center of the line. Figure 3 shows that the convergence time increases exponentially as the source moves away from the line formation. As expected, the line formation is not robust to variations in source location. However, if we limit the field size to a narrow area (i.e. a road with detectors lining along a side), then the performance is good.

#### 6.4.2 Square

We study how the system behaves in a square formation as we increase the field size and as we change the relative position of the detectors and the source in the field.

##### Varying Field Size.

In an empty field of dimension $L$ meters by $L$ meters, we place one detector at each corner and a radiation source in the center of the field (Figure 5(a)). As the field size ($L^2$) increases, the convergence time increases approximately quadratically (Figure 4). As expected, this growth is much better than a line formation.

Figure 4: The convergence time increases quadratically as the field size increases in a square formation.

##### Varying Sensor Positions.

In a fixed field of dimension 300 $\times$ 300 meters, we place four detectors forming a square with sides of length $L$ in the middle of the field, surrounding a radiation source. While keeping the field size constant, we increase the size of the square and measure the time it takes to locate the source (Figure 5(b)). The difference between this experiment and the previous one is that the sensors are not at the corners of the square field.

Figure 5: Experiment setups

The convergence time increases as the detector formation moves further away from the source as shown in Figure 6. Note a local minimum present at $L = 200$ m. The experiment is repeated for various field size, and this local minimum consistently appear at $L = \frac{2}{3}S$, where $S$ is the side length of the field. This phenomenon
occurs because at this distance, the inverse relationship between detector intensity and distance allows the detectors to rapidly rule out the possibility of having a source in the corner.

Figure 6: Convergence time increases as detectors move away from the source. Local minimum occurs at $L = \frac{2}{3} S$.

Varying Source Position.

We study the optimal position for deploying detectors in a square formation in a fixed sized field. In a $50 \times 50$ meters field, four sensors are locked in a square formation of side length $L = [10, 20, 30, 40, 50]$ centered in the field. A radiation source is positioned at a distance $d = [0, 5, 10, 15, 20]$ from the center of the field (Figure 7). We measure the convergence time as $L$ increases and as $d$ increases.

Figure 7: Detectors are placed at the corners of a square of varying size $L$ with a source at varying distance $d$ to the center.

The result shows a general trend of increasing detection time and increasing square size, except when the source is 20 meters away from the center because the source is now approaching the edge of the field. From Figure 8, we can see that the average time reaches a minimum at $L = 20$ meter. It suggests that positioning the detector square at about half the field size is most efficient when the field size is known.

Figure 8: When the detector grid is placed at $L \approx \frac{1}{2} S$. The convergence time averaged over varying source position is minimized.

6.5 Relationship Between Size and Quantity

Next, we study the effect of replacing large detectors with multiple smaller ones. Keeping the total detector surface area constant, $n = [4, 8, 12, 16]$ detectors were arranged in a circle on fields of varying size with length $S = 20, 30, ..., 90, 100, 125, ..., 375, 400$. The circle formed by the detectors is the inscribed circle of the field. Ten trials were performed for each $(n, S)$ pair. We evaluate the result using Criteria 1.

Figure 9: Detectors are placed evenly in a circle that inscribes the field of varying size.

Figure 10(a) shows how the convergence time decreases as we double the number of detectors. Note the dramatic improvement in performance from 4 detector to 8. A close-up at $n = [8, 12, 16]$ is shown in Figure 10(b). This result suggests that it is more beneficial to invest in multiple small detectors than a few large ones.

6.6 Limited Range and Bandwidth

We now consider constraints imposed by wireless communication. Assume the transmitting rate is limited by
Figure 10: The convergence time decreases as each doubling of the detector number while the total surface area remains constant. Note the performance gain has diminishing marginal returns.

100 byte/sec and the transmitting range between peers by 100 meters.

6.7 Study of Group Size

One way to overcome the range limitation is to have the detectors move in tight formations as stated in Algorithm 3. But if so, how big should the group big? Do larger groups always perform better? This experiment studies the relationship between group size and detection efficiency. In a field of 300 \times 300 meters with a radiation source in the center, \( n = [4, 8, 16] \) detectors are placed on the perimeter of a circle of diameter 100 meters at a distance \( d = [0, 25, 50, 75, 100, 125, 150] \) meters away from the source (Figure 11). Any two detectors can communicate with each other in this formation. The result is evaluated using Criteria 1.

Figure 11: A group of \( n \) detectors form a circular formation at a distance \( d \) from the source.

Figure 12 shows that as we increase the number of detectors in the group, the detection time goes down. However, the incremental improvement decreases as the group size gets larger. From \( n = 4 \) to \( n = 8 \), the amount of time for detecting a source 150 meters away is halved. But from \( n = 8 \) to \( n = 16 \), it only improves by less than \( \frac{1}{3} \). In fact, for \( d > 150 \) the stopping criteria is not satisfied for any group size. This result is expected because at a distance, a group of tightly formed detectors can be viewed as one large detector. Thus, the result from Section 6.5 can be generalized to explain this phenomenon, that is, multiple smaller groups are more efficient than one large group.

6.8 Gains from Speed

The performance of a static detector is limited by its distance to the radiation source. If static detectors are ill-positioned and distance to the source is large, detectors may not get a single photon hit over a long period of time, as shown in the previous experiments. Mobile detectors overcome this problem. But will mobile detectors out-perform static ones even under range and bandwidth constraints? How will the performance change when the field size is increased? Will mobility help deal with noise? This experiment studies the performance improvement, if any, from increasing the speed of the detectors in two different size of fields. We test the simple Detection Algorithm 2 and compare it to the naive Algorithm 1 where there is no range or bandwidth limitation. The result is evaluated using Criteria 2.

6.8.1 Small Field

In a field of 300 \times 300 meters, 16 detectors are laid down evenly in a grid formation at time \( t = 0 \) (Figure 13(a)). Initially, each detector is 80 meters away from its nearest neighbors, and can thus communicate with at most 4 detectors, as opposed to 16 when the range and bandwidth are unlimited. The detection time is measured for detector with speed \( v = [0, 0.5, \ldots, 10] \) m/s.

The result is shown in Figure 14(a). At speed \( v = 0 \) m/s (static), detectors with range limitation take 217 seconds to determine the location of the radiation source,
whereas detectors with no limitation take only 173 seconds. The difference in performance between the two diminish for \( v \geq 1 \text{ m/s} \). For \( v \geq 2 \text{ m/s} \), there is almost no gain in performance from increasing the speed in a small field. This result may be due to the simplicity of Algorithm 2, where the detectors do not collaborate essentially.

6.8.2 Large Field

The same experiment in 6.8.1 is repeated in a larger field of 1000 \( \times \) 1000 meters. Initially, each detector is 250 meter away from its nearest neighbors. The result is shown in Figure 14(b). Note the missing data point at \( v = 0 \). Stationary detectors with range limitation are unable to communicate with anyone else and thus the source can never be identified. However, once the detectors start moving, even as slow as 0.5 m/s, this problem is resolved. Also note that in a larger field, where detectors are out of range with each other more frequently than in Section 6.8.1, the detectors with no range limitation consistently perform better by \( \approx 150 \) seconds.

6.9 Effect of Background Noise

The experiment in 6.8.1 is repeated with the presence of a strong uniform background radiation of detector intensity of 1 photon hit every 10 seconds. The result is shown in Figure 14(c). Compared to Figure 14(a), the detection time doubles for each data point for both type of detectors. However, the shape of the curve remains the same, so our findings above are still valid under the influence of background noise.

7. CONCLUSION

This paper evaluates design tradeoffs in designs of intelligent radiation sensor systems constructed by fusing data from multiple wireless-enabled, static and mobile intelligent personal radiation locators. The development of IRSS from existing and planned IPRLs is important for the nation’s security. IRSS are ideal examples of information processing in sensor networks. Our group at Caltech has been working with partners on designing and implementing IPRLs; the research reported in this paper explores basic questions about potential benefits from integrating information from multiple IPRLs. The analysis presented here deals with fundamental issues such as understanding the decrease in time to locate a radiation source as a function of the number of sensors, speeds of mobile sensors, and tradeoffs of more small sensors versus few large sensors. Understanding these basic canonical problems gives insight that helps in deal-

Figure 13: 16 detectors are placed evenly in the field initially.

Figure 14: Mobile detectors with communication range and bandwidth limitation
ing with more complex problems such as the optimum strategies to be used by maritime boarding parties that have to deal with the photon-shielding effects of ships’ metal structures and evasive action taken by traffickers.

The experiments show that networks of collaborating IPRLs locate threats much faster than non-collaborating IPRLs. Thus, our experiments make a strong case for IRSS. The analysis also demonstrates that networks of mobile sensors are much more effective than networks of stationary sensors. Our analysis shows that there are benefits to increasing the number of sensors with diminishing marginal returns with the addition of each sensor. Likewise, the time to detect a source decreases with the speed of mobile sensors, but here too there is diminishing marginal returns. An important finding is that by moving, using the simplest algorithm, at a speed as slow as 0.5 m/s, and under the range and bandwidth constraints with the presence of background noise, the unsolvable detection problem becomes solvable. This result strengthens the idea of replacing large portal style sensor with mobile handheld sensor network carried by security personnel. The analysis and programs described here can be used to determine configurations of networks of static and mobile sensors most appropriate to given problems.

Further work needs to be carried out on realistic scenarios such as protecting people in stadiums and public malls from dirty bombers, and in interdicting nuclear material from ships and at ports and airports. Suites of experiments should be carried out on evasive action that may be taken by people carrying nuclear material. Gaming environments that help security people play roles of terrorists and first responders will help in dealing with evasive action[8]. Finally, IRSS will have to be tested in the field to evaluate their effectiveness in realistic situations.

8. REFERENCES


APPENDIX

A. PARAMETRIC UPDATE USING A PRIOR DISTRIBUTION OF SOURCE INTENSITY

The modified Bayesian Update method for intensity estimation is as follows. Initially we choose a distribution of $[\mu_1, \mu_2, \ldots, \mu_n]$ with uniform probability that sums to 1. The range of $\mu_i$ is based on informed guess of the source intensity. Of each $\mu_i$, we apply Equation 5 to get the a posteriori distribution.

$$\frac{p_{t}^{\mu}}{p_{t}^{\mu}} = \prod_{j=1}^{J} f(n_{j}, \lambda_{jk}, \Delta t) \quad (7)$$

The resulting probability distribution map is the $n$ maps superimposed onto one with $p_j^i$ being the sum of all $p_j^{\mu}$.

$$p_j^i = \sum_{i=1}^{n} p_j^{\mu} \quad (8)$$

After normalizing using Equation 4, we get the final $p_j^i$. We can also examine the a posteriori distribution of $\mu_i$. The probability of a specific $\mu_i$ being the real source intensity is calculated.

$$P(\mu_i = \mu) = \frac{\sum_{k=1}^{K} p_{jk}^i}{\sum_{k=1}^{K} p_{jk}^i} \quad (9)$$

After a few iterations of the above process, the a posteriori distribution in intensity and location both converge.

B. NON-PARAMETRIC UPDATE METHOD

We can define $p_j(x, y)$ as the conditional probability that detector $D_j$ detects a photon emitted by a source at location $(x, y)$ given that one of the $D$ detectors detects a photon from the same location.

$$p_j(x, y) = \frac{\lambda_j(x, y)}{\sum_{j=1}^{D} \lambda_j(x, y)} \quad (10)$$

Let $d_j(x, y)$ be the distance from detector $j$ to location $(x, y)$. Combined with Equation 2 we find that $p_j(x, y)$ is inverse proportional to $d_j(x, y)$

$$p_j(x, y) \propto \frac{1}{d_j^2(x, y)} \quad (11)$$

We apply Equation 11 to all grids $(x_k, y_k)$. The result is a rapid convergence in a posteriori distribution to the detector positions that are closest to the source. However, it does not pin-point down the exact location of the source. This result is anticipated for lack of information in source intensity.

C. MAXIMUM LIKELIHOOD ESTIMATOR FOR SOURCE INTENSITY AND LOCATION

The frequency of photons received by a radiation detector is given by Equation 1. In the presence of multiple detectors, the combined probability becomes:

$$\Pi_{k=1}^{K} P(n_k | \mu, t) = \Pi_{k=1}^{K} \frac{\lambda_k^{n_k} e^{-\lambda_k}}{n_k!} \quad (12)$$

Where $n_k$ is the number of photon received by detector $D_k$ in time $t$. $\lambda_k$ is the intensity at detector $D_k$ and is related to the source intensity $\mu$ by Equation 2.

We can measure $n_k$ for $1 \leq k \leq K$ and $d_k$, which is the distance from detector $D_k$ to the source. Given this information, the correct estimate of source intensity and location will maximize this probability as well as the logarithm of it.

$$\ln(\Pi_{k=1}^{K} P(n_k | \mu, t))$$

$$= \ln(\Pi_{k=1}^{K} \frac{e^{-\lambda_k} \lambda_k^{n_k}}{n_k!})$$

$$= \sum_{k=1}^{K} \ln(\frac{e^{-\lambda_k} \lambda_k^{n_k}}{n_k!})$$

$$= \sum_{k=1}^{K} -\lambda_k + n_k \ln(\lambda_k) - \ln(n_k!)$$

$$= \sum_{k=1}^{K} -\frac{\mu A \Delta t}{4\pi d_k^2} + n_k \ln(\frac{\mu A \Delta t}{4\pi d_k^2}) - \ln(n_k!)$$

Let $\mu_{MLE}$ and $x_{MLE}$ be the source intensity and location that optimize this target function. $\mu_{MLE}$ is related to $x_{MLE}$ by maximizing this function with respect to $\mu$ first.

$$\frac{d}{d\mu} \ln(\Pi_{k=1}^{K} P(n_k | \mu, t)) = \sum_{k=1}^{K} -\frac{A \Delta t}{4\pi d_k^2} + \frac{n_k}{\mu} = 0 \quad (13)$$

We get,

$$\frac{1}{\lambda} \sum_{k=1}^{K} n_k = \sum_{k=1}^{K} \frac{A \Delta t}{4\pi d_k^2} \quad (14)$$

After simplifying,

$$\mu_{MLE} = \frac{\sum_{k=1}^{K} n_k}{\sum_{k=1}^{K} \frac{A \Delta t}{4\pi d_k^2}} = \frac{4\pi \sum_{k=1}^{K} n_k}{A \sum_{k=1}^{K} \frac{A \Delta t}{4\pi d_k^2}} \quad (15)$$

This relationship allows us to reduce the variables to be maximized to one $(x)$. After $x_{MLE}$ is found, we can use this relationship to calculate $\mu_{MLE}$.